Name ________________________ ID3 _ _ _

ECS 332: Principles of Communications **2017/1**

HW 6 — Due: Nov 17, 4 PM

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) This assignment has 6 pages.

(b) (1 pt) Work and write your answers **directly on these provided sheets** (not on other blank sheet(s) of paper). Hard-copies are distributed in class.

(c) (1 pt) Write your first name and the last three digits of your student ID on the upper-right corner of this page.

(d) (8 pt) Try to solve all non-optional problems.

(e) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. Find the instantaneous frequency of the signal $g(t) = 3\sqrt{2}\cos\left(\overbrace{12t^3 + t^2}^{\theta(t)}\right)$

(a) at time $t = 0$

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t) = \frac{1}{2\pi}\left(36t^2 + 2t\right)$$

$$f(0) = \frac{1}{2\pi}\left(36 \times 0^2 + 2 \times 0\right) = 0$$

(b) at time $t = 2$

$$f(2) = \frac{1}{2\pi}\left(36 \times 2^2 + 2 \times 2\right) = \frac{1}{2\pi}\left(\quad\right) = ?$$

Problem 2. Recall that, in QAM system, the transmitted signal is of the form

$$x_{\mathrm{QAM}}(t) = m_1(t)\sqrt{2}\cos(2\pi f_c t) + m_2(t)\sqrt{2}\sin(2\pi f_c t).$$

We want to express x_{QAM} in the form

$$x_{\mathrm{QAM}}(t) = \sqrt{2}E(t)\cos(2\pi f_c t + \phi(t)),$$

where $E(t) \geq 0$ and $\phi(t) \in (-180°, 180°]$. (This shows that QAM can be expressed as a combination of amplitude modulation and phase modulation.)

Consider $m_1(t)$ and $m_2(t)$ plotted in Figure 6.1.
Draw the corresponding $E(t)$ and $\phi(t)$.

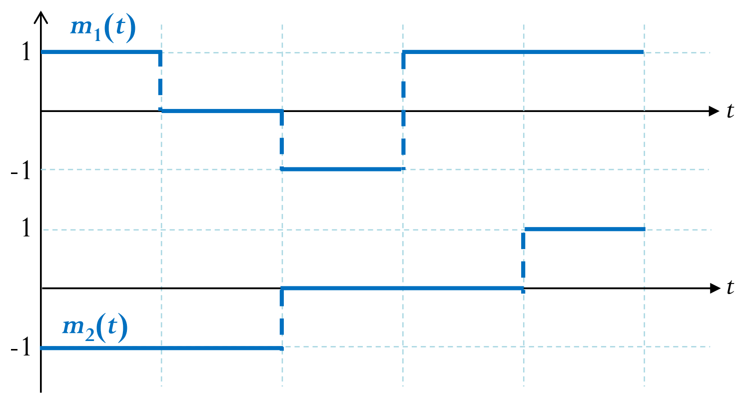

Figure 6.1: $m_1(t)$ and $m_2(t)$ for Problem 2

Problem 3. Consider the message $m(t)$ along with the carrier signal $\cos(2\pi f_c t + \phi)$ plotted in Figure 6.2. Note that $m(1.5) = 40$.

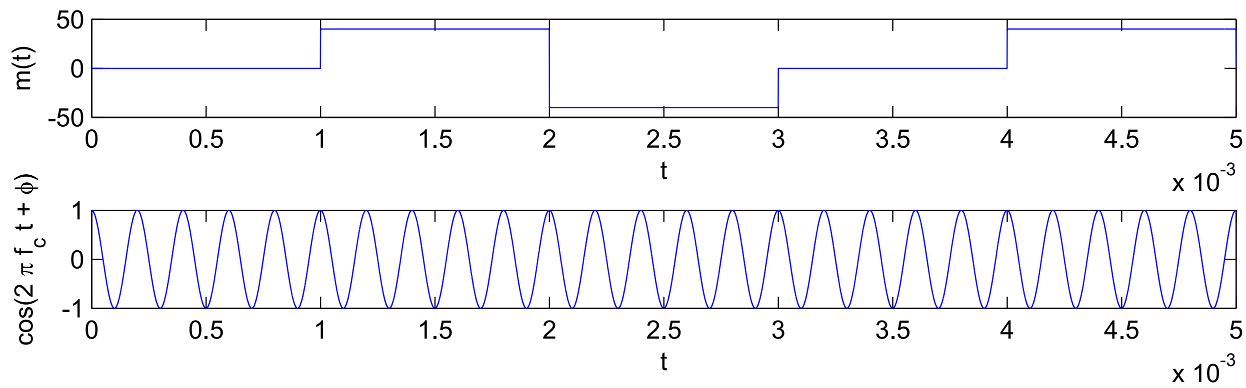

Figure 6.2: The message and the carrier signals for Problem 3.

(a) Find the carrier frequency f_c from the plot. (Hint: It is an integer.)

(b) Sketch the following signals. Make sure that (the unspecified parameter(s) are selected such that) the important "features" of the graphs can be seen clearly.

(i) $x_{\mathrm{AM}}(t) = (A + m(t))\cos(2\pi f_c t + \phi)$ whose modulation index $\mu = 100\%$.

(ii) $x_{\mathrm{FM}}(t) = A\cos\left(2\pi f_c t + \phi + 2\pi k_f \int\limits_{-\infty}^{t} m(\tau)d\tau\right)$.

- You may assume $m(t) = 0$ for $t < 0$.

(iii) $x_{\text{PM}}(t) = A\cos\left(2\pi f_c t + \phi + k_p m(t)\right)$ with $k_p = \frac{\pi}{m_p}$.

Problem 4. Consider the FM transmitted signal

$$x_{\text{FM}}(t) = A\cos\left(2\pi f_c t + \phi + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\right),$$ $: f(t) = f_c + k_f m(t)$

where $f_c = 5$ [kHz], $A = 1$, and $k_f = 75$. The message $m(t)$ is shown in Figure 6.3.

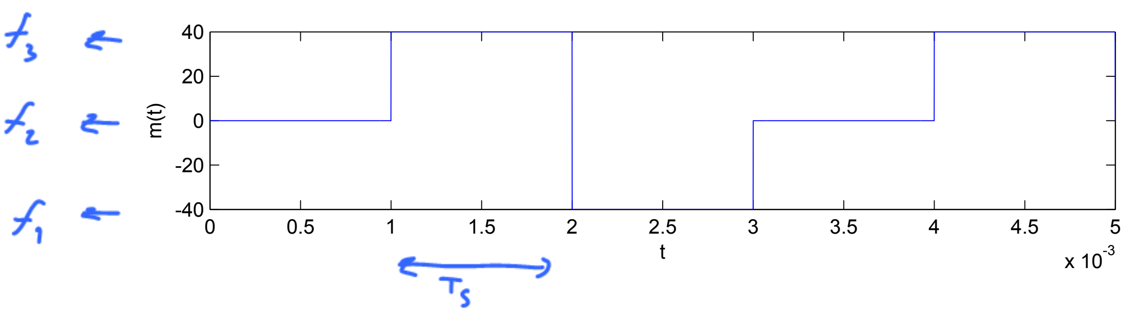

Figure 6.3: The message $m(t)$ for Problem 4

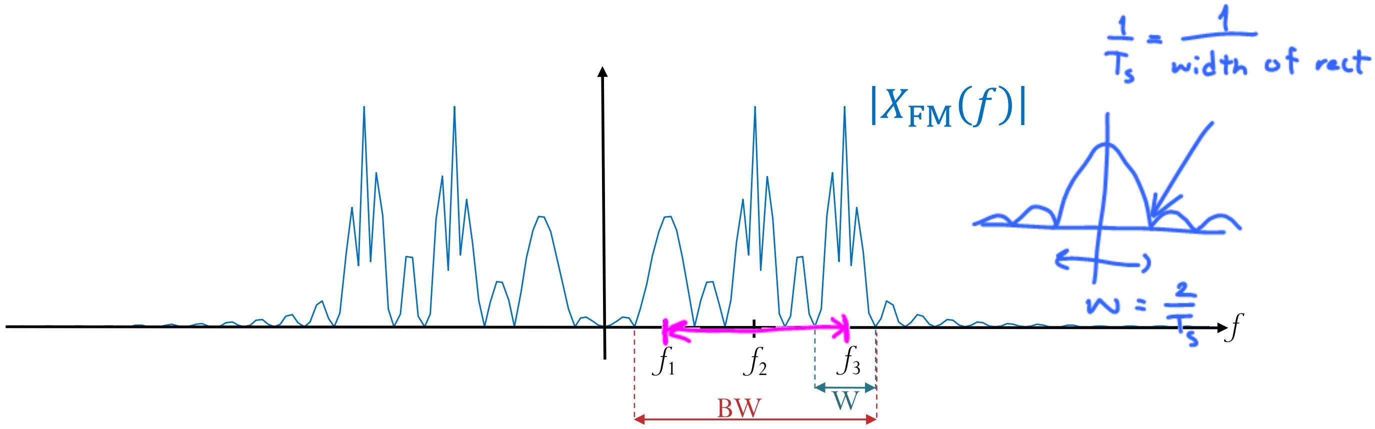

Figure 6.4: The magnitude spectrum $|X_{\text{FM}}(f)|$ for Problem 4

The magnitude spectrum $|X_{\mathrm{FM}}(f)|$ is plotted in Figure 6.4.

(a) Find the values of f_1, f_2, and f_3.

$f_1 = f_c + k_f(-40) = 5\times10^3 + 75(-40) =$

$f_2 =$

$f_3 =$

(b) Find the width W in Figure 6.4.

$W = \frac{2}{T_s} = \frac{2}{1\times10^{-3}} = 2\ \mathrm{kHz}$

(c) Find the occupied bandwidth denoted by BW in Figure 6.4.

$(f_3 - f_1) + W =$

Extra Questions

Here are some optional questions for those who want more practice.

Problem 5. Recall that, in QAM system, the transmitted signal is of the form

$$x_{\mathrm{QAM}}(t) = m_1(t)\sqrt{2}\cos(2\pi f_c t) + m_2(t)\sqrt{2}\sin(2\pi f_c t).$$

In class, we have shown that

$$\mathrm{LPF}\left\{x_{\mathrm{QAM}}(t)\sqrt{2}\cos(2\pi f_c t)\right\} = m_1(t).$$

Give a similar proof to show that

$$\mathrm{LPF}\left\{x_{\mathrm{QAM}}(t)\sqrt{2}\sin(2\pi f_c t)\right\} = m_2(t).$$

Problem 6. In ***quadrature amplitude modulation*** (***QAM***) or ***quadrature multiplexing***, two baseband signals $m_1(t)$ and $m_2(t)$ are transmitted simultaneously via the following QAM signal:

$$x_{\mathrm{QAM}}(t) = m_1(t)\sqrt{2}\cos(\omega_c t) + m_2(t)\sqrt{2}\sin(\omega_c t).$$

An error in the phase or the frequency of the carrier at the demodulator in QAM will result in loss and interference between the two channels (cochannel interference).

In this problem, show that

$$\mathrm{LPF}\left\{x_{\mathrm{QAM}}(t)\sqrt{2}\cos\left((\omega_c+\Delta\omega)t+\delta\right)\right\} = m_1(t)\cos\left((\Delta\omega)t+\delta\right) - m_2(t)\sin\left((\Delta\omega)t+\delta\right)$$
$$\mathrm{LPF}\left\{x_{\mathrm{QAM}}(t)\sqrt{2}\sin\left((\omega_c+\Delta\omega)t+\delta\right)\right\} = m_1(t)\sin\left((\Delta\omega)t+\delta\right) + m_2(t)\cos\left((\Delta\omega)t+\delta\right).$$

Problem 7. As in Problem 2, in QAM system, the transmitted signal is of the form

$$x_{\mathrm{QAM}}(t) = m_1(t)\sqrt{2}\cos(2\pi f_c t) + m_2(t)\sqrt{2}\sin(2\pi f_c t).$$

We want to express x_{QAM} in the form

$$x_{\mathrm{QAM}}(t) = \sqrt{2}E(t)\cos(2\pi f_c t + \phi(t)),$$

where $E(t) \geq 0$ and $\phi(t) \in (-180°, 180°]$.

In each part below, we consider different examples of the messages $m_1(t)$ and $m_2(t)$.

(a) Suppose $m_1(t) = \cos(2\pi Bt)$ and $m_2(t) = \sin(2\pi Bt)$ where $0 < B \ll f_c$. Find $E(t)$ and $\phi(t)$. Hint: $\cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\alpha-\beta)$

(b) Suppose $m_1(t) = \cos(2\pi Bt)$ and $m_2(t) = 2\sin(2\pi Bt)$. Let $f_c = 5$ and B = 2. Use `MATLAB` to plot the corresponding $E(t)$ and $\phi(t)$ from $t = 0$ to $t = 5$ [sec]. (Hint: the function `angle` or `atan2` will be helpful here.)

Problem 8. Consider a complex-valued signal $x(t)$ whose Fourier transform is $X(f)$.

(a) Find and simplify the Fourier transform of $x^*(t)$.

(b) Find and simplify the Fourier transform of $\mathrm{Re}\{x(t)\}$.

- Hint: $x(t) + x^*(t) = ?$

Problem 9. Consider a (complex-valued) baseband signal $x_b(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} X_b(f)$ which is band-limited to B, i.e., $|X_b(f)| = 0$ for $|f| > B$. We also assume that $f_c \gg B$.

(a) The passband signal $x_p(t)$ is given by

$$x_p(t) = \sqrt{2}\,\mathrm{Re}\left\{e^{j2\pi f_c t}x_b(t)\right\}.$$

Find and simplify the Fourier transform of $x_p(t)$.

(b) Find and simplify

$$\mathrm{LPF}\left\{\sqrt{2}\left(\underbrace{\sqrt{2}\,\mathrm{Re}\left\{e^{j2\pi f_c t}x_b(t)\right\}}_{x_p(t)}\right)e^{-j2\pi f_c t}\right\}.$$

Assume that the frequency response of the LPF is given by

$$H_{LP}(f) = \begin{cases} 1, & |f| \leq B \\ 0, & \text{otherwise.} \end{cases}$$